THE CALL OF MATHEMATICS

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Abstract. A few remarks on how mathematics quests for freedom.

Mathematics prevails in knowledge as the most ancient of sciences. However, in the beginning was the word. We must remember that the olden "logos" resides beyond grammar. Today's mathematics became the bastion of logic, the savior of the order of mind and the objectivity of reasoning.

The intellectual field resides beyond the grips of the law of diminishing returns. The more we know, the huger become the frontiers with the unbeknown, the oftener we meet the mysterious. The twentieth century enriched our geometrical views with the concepts of space-time and fractality. Each instance of knowledge is an event, a point in the Minkowski 4-space. The realm of our knowledge comprises a clearly bounded set of these instances. The frontiers of science produce the boundary between the known and the unknown which is undoubtedly fractal and we have no grounds to assume it rectifiable or measurable. It is worth noting in parentheses that rather smooth are the routes to the frontiers of science which are charted by teachers, professors, and all other kinds of educationalists. Pedagogics dislikes saltations and sharp changes of the prevailing paradigm. Possibly, these topological obstructions reflect some objective difficulties in modernizing education. The proofs are uncountable of the fractality of the boundary between the known and the unbeknown. Among them we see such negative trends as the unleashed growth of pseudoscience, mysticism, and other forms of obscurantism which creep into all lacunas of the unbeknown. As revelations of fractality appear the most unexpected, beautiful, and stunning interrelations between seemingly distant areas and directions of science. Mathematics serves as the principal catalyst of the unity of science. There is evidence galore of the indispensability of mathematics in modernization and sustainable development.

We are granted the blissful world that has the indisputable property of unique existence. The solitude of reality was perceived by our ancestors as the ultimate proof of unicity. Mathematics has never liberated itself from the tethers of experimentation. The reason is not the simple fact that we still complete proofs by declaring "obvious." Alive and rather popular are the views of mathematics as a toolkit for the natural sciences. These stances may be expressed by the slogan "mathematics is experimental theoretical physics." Not less popular is the dual claim "theoretical physics is experimental mathematics." This coupled mottoes reflect the close affinity of the trails of thought in mathematics and the natural sciences.

It is worth observing that the dogmata of faith and the principles of theology are also well reflected in the history of mathematical theories. Variational calculus was

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invented in search of better understanding of the principles of mechanics, resting on the religious views of the universal beauty and harmony of the act of creation.

Mathematics is a rather specific area of intellectual creativity which possess its own unparallel particularities. Georg Cantor, the founder of set theory, wrote in one of his classical papers in 1883 as follows: "...das *Wesen* der *Mathematik* liegt grerade in ihrer *Freiheit*." In other words, "the essence of mathematics resides in its freedom." The freedom of modern mathematics does not reduce to the absence of exogenous limitations of the objects and methods of research. To a great extent, the freedom of mathematics is disclosed in the new intellectual tools it provides for taming the universe, liberating a human being, and expanding the boundaries of his or her independence.

The twentieth century marked an important twist in the content of mathematics. Mathematical ideas imbued the humanitarian sphere and, primarily, politics, sociology, and economics. Social events are principally volatile and possess a high degree of uncertainty. Economic processes utilize a wide range of the admissible ways of production, organization, and management. The nature of nonunicity in economics transpires: The genuine interests of human beings cannot fail to be contradictory. The unique solution is an oxymoron in any nontrivial problem of economics which refers to the distribution of goods between a few agents. It is not by chance that the social sciences and instances of humanitarian mentality invoke the numerous hypotheses of the best organization of production and consumption, the justest social structure, the codices of rational behavior and moral conduct, etc.

The twentieth century became the age of freedom. Plurality and unicity were confronted as collectivism and individualism. Many particular phenomena of life and culture reflect their distinction. The dissolution of monarchism and tyranny were accompanied by the rise of parliamentarism and democracy. In mathematics the quest for plurality led to the abandonment of the overwhelming pressure of unicity and categoricity. The latter ideas were practically absent, at least minor, in Ancient Greece and sprang to life in the epoch of absolutism and Christianity. Quantum mechanics and Heisenberg's uncertainty incorporated plurality in physics. The waves of modernism in poetry and artistry should be also listed. Mankind had changed all valleys of residence and dream.

The thesis of universal mathematization enlightens many trends of today's thought. Many new synthetical areas of research are the gains of mathematics which are decorated with outstanding advances in economical cybernetics, theoretical programming, mathematical linguistics, mathematical chemistry, and mathematical biology. Mathematization of the human sciences and the human dimension of the natural sciences are familiar features of modernism.

Mathematics is a human science involving the abstractions in which the human beings perceive forms and relations. Mathematics is impossible without the disciples, professional mathematicians. Obviously, the essence of mathematics is disclosed to us only as expressed in the contributions of scientists. Therefore, it would be not a great exaggeration to paraphrase the words of Cantor and say that *the essence of a mathematician resides and reveals itself in his or her freedom.*

In science we appraise and appreciate that which makes us wiser. The notions of a good theory open up new possibilities of solving particular problems. Rewarding is the problem whose solution paves way to new fruitful concepts and methods. Condescension is the mother of mediocrity. A fresh product of a mediocrity is called

a banality. Time makes banal the most splendid achievements, seminal theories, and challenging problems. Indispensability is the most important quality of a good problem or theory which refrains us from producing banalities.

The greatest minds create indispensable scientific concepts and ponder them over. They pose indispensable scientific problems and contemplate over their solutions. The indispensable theories and problems propel science. The best scientists propounded not only indispensable theories and addressed not only indispensable problems. But only indispensable theorems and problems make these scientists great.

A good theory enables us to settle some indispensable problems. We know many classical examples of fruitful and powerful theories. Euclidean geometry and differential calculus were gigantic breakthroughs in the understanding and mastering the reality. Centuries witness the strength and power of these theories yielding everyday's solutions of uncountably many practical problems. Solution of an indispensable problem is a grind stone for a good theory since it requires a new conceptual technique and revision of the available theoretical gadgets. Squaring the circle, the variational principles of mechanics, and the majority of the Hilbert problems provide examples of the questions that brought about sweeping changes in the theoretical outlooks of science.

We must not narrow and simplify the concept of a problem. Science endeavors to make the complex the simple. Therefore, always actual are the reconsideration and inventory of the available theories as well as their simplification, generalization, and unification. The history of science knows many examples of the perfection, beauty, and practical power of the theories that arose by way of abstraction and codification of the preceding views. The success of a new theory proves that this theory was indispensable.

Freedom in science is the consciousness and appreciation of the indispensable, a vaccine against banality. The call of freedom is inseparable from the call of mathematics.

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